a diminution in $\partial Y/\partial x_s$ of not more than 15%. In this case the system (11)-(12) is easily solved graphically, to give the domain of allowable changes in x_2 and its corresponding concentrations (Fig. 3). The allowable interval of the cuvette temperature change is determined easily from (10): $x_1 = 1-0.35$, which corresponds to T = 45.2 ± 4.8°C in natural variables. Therefore, the operating regime found for the thermooptical gas analyzer requires ±10% accuracy in maintaining the magnitude of the flow rate and the heating temperature.

NOTATION

T, temperature; q, heat flux; R, channel radius; θ, r, z, coordinates; $\bar{r} = r/R$; ε_0, dielectric permittivity of the gas; σ, arc length; $\tau = \int_0^\sigma \varepsilon^{-1/2} d\sigma$; $\bar{\tau} = \tau/R$; λ, heat-conduction coefficient; Re, Reynolds number; Pr, Prandtl number; x_i, coordinates in factor space.

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EXISTENCE OF A CLASSICAL VARIATIONAL PRINCIPLE FOR NONLINEAR COUPLED HEAT AND MASS TRANSPORT

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Using the example of nonlinear coupled heat and mass transport, we examine whether a functional exists for which the required kinetic equations follow from the condition that the functional be stationary.

In nonlinear problems of heat conduction and coupled heat and mass transport, variational methods are widely used today. These methods are based on variational formulations of the problem, called in physics variational principles.

In the restricted sense, by a variational principle we mean the statement that a certain functional must attain a maximum or minimum [1]. This functional contains all of the defining equations and boundary conditions for the problem. Thus, the equations and boundary conditions follow from the variational formulation as conditions that the functional be stationary (the Euler equations). We will refer to this kind of variational formulation as a classical variational principle. Examples include Hamilton's principle of least action in mechanics [2], Castilyan's principle in the theory of elasticity [3], Fermat's principle in optics [4], and certain variational principles in the classical and relativistic theory of fields.

The classical variational formulations are distinguished by simplicity, generality, elegance, and are of heuristic value as well. However, attempts at obtaining these variational principles by fitting a variational equation to a problem previously formulated in differential form is difficult and not always successful. This is because many differential equations and systems of equations, especially nonlinear ones, do not have classical variational principles.

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Since nonlinear problems in engineering physics are important in applications, we discuss a method of determining the existence of a classical variational principle for this type of problem. Specifically, we consider the example of nonlinear unsteady coupled heat and mass transport.

We consider coupled heat and mass transport in a medium of volume v, bounded by a surface S. Taking c_q, c_m, γ, ρ = const, we have

$$\lambda_q = \lambda_q^* F_1(\vartheta_1, \vartheta_2), \quad \lambda_m = \lambda_m^* F_2(\vartheta_1, \vartheta_2), \quad \varepsilon = \varepsilon^* F_3(\vartheta_1, \vartheta_2),$$
$$\delta = \delta^* F_4(\vartheta_1, \vartheta_2).$$

In this case the system of equations for coupled heat and mass transport [5] can be put in the form

$$\Theta_{1,4} = (F_1\Theta_{1,\alpha})_{,\alpha} - \mathrm{Ko}^* F_3\Theta_{2,4},$$
$$\Theta_{2,4} = -\mathrm{Lu\,Pn}\,(F_2F_4\Theta_{1,\alpha})_{,\alpha} + \mathrm{Lu}\,(F_2\Theta_{2,\alpha})_{,\alpha}, \tag{1}$$

and the boundary conditions are written as

$$\Theta_i(P, 0) = M_i(P) \quad (P \in v), \tag{2}$$

$$\Theta_i(P, t) = \Phi_{i1}(P, t) \quad (P \in S_1), \tag{3}$$

$$n_\alpha\Theta_{i,\alpha}(P, t) = \Phi_{i2}(P, t) \quad (P \in S_2), \tag{4}$$

$$n_\alpha\Theta_{i,\alpha}(P, t) = \Phi_{i3}[\Theta_1(P, t), \Theta_2(P, t)] \quad (P \in S_3), \tag{5}$$

where $S_1 \cup S_2 \cup S_3 = S$; M_i, Φ_i, ν are known functions.

Equation (2) is the initial condition, (3) and (4) are boundary conditions of the first and second kinds, and (5) is a boundary condition of the third kind.

In order to determine whether a classical variational principle exists for equations (1)-(5), we use the general approach of [6]. Let

$$N(\Theta) = 0 \tag{6}$$

represent the differential statement of the problem. The operator N is called potential if there exists a functional (potential) I such that

$$\mathrm{grad}\, I = N(\Theta). \tag{7}$$

The differentiation on the left-hand side of (7) is assumed to be in the sence of a Gato derivative (the differentiation of a functional I(Θ) is defined as [7] $I_\lambda(\Theta + \lambda\psi)|_{\lambda=0}$ and this reduces to the definition of the variation δI). Then the expression grad I = 0 gives the Euler equations, where the functional is represented as [6]

$$I(\Theta) = \int_\Omega \Theta \int_0^1 N(\lambda\Theta)\, d\lambda d\Omega, \tag{8}$$

so that the variational principle has the form δI = 0. Here λ is a scalar and Ω is the physical region under consideration (the region where the functions Θ are defined). Whether N is potential or not depends on whether the Gato derivative is symmetric or not. If the Gato derivative is symmetric, then a classical variation principle follows at once in form (8). The symmetry of the Gato derivative can be tested in our case with the help of certain conditions. A necessary condition for the Gato derivative to be symmetric is that the leading derivative in the equations [6] be even. We see immediately that the boundary conditions (2)-(5) do not satisfy this requirement and therefore they cannot be derived from the functional (8) as boundary conditions for the problem. Equation (1) has even-order leading derivatives, hence we examine the possibility of constructing a functional (8) for these equations without the boundary conditions. With the help of elementary transformations, we write system (1) in the form N = 0 where N = (N_1, N_2) and

$$N_1 = \Theta_{1,4} - (F_{1,\Theta_1}\Theta_{1,\alpha} + F_{1,\Theta_2}\Theta_{2,\alpha})\,\Theta_{1,\alpha} - F_1\Theta_{1,\alpha\alpha} + \mathrm{Ko}^* F_3\Theta_{2,4},$$
$$N_2 = \Theta_{2,4} + \mathrm{Lu\,Pn}\,\{[(F_2F_4)_{,\Theta_1}\Theta_{1,\alpha} + (F_2F_4)_{,\Theta_2}\Theta_{2,\alpha}]\,\Theta_{1,\alpha} + F_2F_4\Theta_{1,\alpha\alpha}\} - \mathrm{Lu}\,[(F_{2,\Theta_1}\Theta_{1,\alpha} + F_{2,\Theta_2}\Theta_{2,\alpha})\,\Theta_{2,\alpha} + F_2\Theta_{2,\alpha\alpha}]. \tag{9}$$

Now we can use the condition obtained in [8] for a system of nonlinear second-order differential equations: a functional in the form of expression (8) exists and Eq. (1) will correspond to the Euler equations if the following condition is satisfied:

$$N_{m,\Theta_{i,jk}} = N_{i,\Theta_{m,jk}},$$

$$N_{m,\Theta_{i,j}} = -N_{i,\Theta_{m,j}} + 2(N_{i,\Theta_{m,jk}})_{,k}, \qquad (10)$$

$$N_{m,\Theta_i} = N_{i,\Theta_m} - (N_{i,\Theta_{m,j}})_{,j} + (N_{i,\Theta_{m,jk}})_{,jk}.$$

In order to examine the third condition in (10), one must have specific forms for the functions F_1, F_2, F_3, F_4. However, this is not necessary in our case because the first and second conditions in (10) give a negative answer to the question. Indeed, the first of these equations for N_i in the form (9) is not satisfied for j = k = 1, 2, 3 and the second is not satisfied for all values of j.

Thus, a classical variational principle does not exist for the complete boundary-value problem of coupled heat and mass transport (1)-(5), nor for the system of equations (1) considered separately. This negative result does not rule out the possibility of using variational methods in calculating the solution to (1)-(5). These methods could be based on a variational principle of a nonclassical type. Examples of such variational equations are given in [9-11].

NOTATION

$(\ )_{,\alpha}$ (α = 1, 2, 3), partial derivative with respect to the spatial coordinate x_α; $(\ )_{,4}$, partial derivative with respect to the generalized time Fo; $(\ )_{,\Theta_i}$ partial derivative with respect to Θ_i (i = 1, 2); $(\ )^*$, characteristic dimensional factors; $P = P(x_1, x_2, x_3)$, field point; t, time; n_α, components of a unit vector along the outward normal to the bounding surface S; λ_q, λ_m, c_q, c_m, thermal conductivity, mass conductivity, heat capacity and mass capacity, respectively; ε, phase transition criterion; ρ, specific heat of phase transition; γ, density of a perfectly dry body; ϑ_1, ϑ_2 temperature and mass-transport potential; $Pn = \delta^*\vartheta_{1S}/\vartheta_{2S}$, Posnov number; $Ko^* = \varepsilon^*\rho(c_m/c_q)\,\vartheta_{2S}/\vartheta_{1S}$, modified Kossovich number; $Lu = a_m/a_q$, Lykov number; $Fo = a_q t$, generalized time; $a_q = \lambda_q^*/c_q\gamma$; $a_m = \lambda_m^*/c_m\gamma$; $\Theta_1 = \vartheta_1/\vartheta_{1S}$, dimensionless temperature; $\Theta_2 = \vartheta_2/\vartheta_{2S}$, dimensionless mass-transport potential; repeated Greek indices are summed from 1 to 3; repeated Latin indices are summed from 1 to 4; free Latin indices take values 1 and 2; free Greek indices take values 1, 2, 3.

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